Modeling the Business Starting with Shortages for Deteriorating Items

Uttam Kumar Khedlekar

Department of Mathematics and Statistics, Sagar Central University,

Sagar, Madhya Pradesh, India-470003.

uvkkcm@yahoo.co.in

Abstract

In this paper, we have solved a problem occurring in those businesses starting with shortages. Management has the opportunity to order the stock according to customer's response. Deterioration factor has been incorporated along with time dependent demand. A theory has been developed to obtain the optimal solution to the proposed problem with valuable conclusions. A numerical example was appended and simulation study was carried out to illustrate the proposed model with managerial insights.

Keywords

Inventory; Time dependent demand; Deterioration; Shortage; Optimal time

Subject Classification: *90 B 50, 90 B 30, 90 B 05*

Introduction

A business could be started with no stock of items shortage as advance booking of products and could be supplied after a duration. For example, advance booking of LPG gas, electricity supply and pre public offer of equity share before proper functioning the company. We considered linear demand with assumption that the business starts with no inventory. From beginning, the classical inventory model and many other models are presented by researchers, but no model exists in literature that considered the shortage in begining. Few items in the market are of high need for people like sugar, wheat, oil whose shortage break the customer's faith and arrival pattern. This motivates retailers to order for excess units of item for maintaining the inventory in spite of being deterioration. Moreover, deterioration is manageable for many items by virtue of modern advanced storage technologies. Inventory model presents a real life problem (situation) which helps to run the business smoothly.

Burwell *et al.* (1997) solved the problem arising in business by providing freight discounts and presenting economic lot size model with price-dependent demand for deteriorating item. You (2005), and Khedlekar and Shukla (2013) investigated the effect of price reduction on market demand of a product with price sensitive demand while Joglekar (2008) investigated the effect of price increase on optimal policy. The both strategies are not contradicted to each other but the first one is for deckling market and the other one is for growing market.

Shin (1997) determined an optimal policy for retail price and lot size under day-term supplier credit. Shukla and Khedlekar (2014) introduced a concept of convertible items which converts it into two consecutive converted items and optimises the cost for conversion. Matsuyama (2001) presented a general EOQ model considering holding costs, unit purchase costs, and setup costs that are time-dependent and continuous general demand functions. The problem has been solved by dynamic programming so as to find ordering point, ordering quantity, and incurred costs.

The research overviewing Emagharby and Keskinocak (2003) is to determine the dynamic pricing and order level in deckling market. Teng and Chang (2005) presented an economic production quantity (EPQ) model for deteriorating items when the demand rate depends not only on the on-display stock, but also on the selling price per unit. The manipulation in selling price is the best policy for the organization as well as for the customers.

Wen and Chen (2005) suggested a dynamic pricing policy for selling a given stock of identical perishable products over a finite time horizon on the internet. The sale ends either when the entire stock is sold out, or when the deadline is over. Here, the objective of the seller is to find a dynamic pricing policy that maximizes the total expected revenues. The EOQ model designed by Hou and Lin (2006) reflects how a demand pattern in which price, time, and stock

dependent affects the discount in cash. They discussed an EOQ inventory model which took into account the inflation and time value of money of the stock-dependent selling price. Existence and uniqueness of the optimal solution has not been shown in that article.

Lai *et al.* (2006) algebraically approached the optimal value of cost function rather than the traditional calculus method and modified the EPQ model earlier presented by Chang (2004) in which he considered variable lead time with shortages. Recent contribution in EPQ models is a source of esteem importance like Birbil *et al.* (2007) and Hou (2007) Shukla *et al.* (2010*a*), Khedlekar (2012), Kedlekar *et al.* (2013 *a*&*b*, 2014), Bhaskaran *et al.* (2010), Kumar (2012*a*), Kumar (2012*b*) and Kumar (2012*c*). Motivation is derived due to Wu (2002) and Shukla *et al.* (2010*b*) for considering the shortages in beginning in a business and developed the model described below.

Assumptions and Notations

Suppose that a business start with shortage of a product and demand is $D(t)=(a+bt)$ and shortage accumulats till time t_1. Company receives order by the vendor at time t_1 and thus shortage immediate fulfilled and inventory reaches up to level $I_2(t_1)$. The inventory level $I_2(t_1)$ is sufficient to fulfil the demand till time T. We look for optimal time t_1 (optimal time for accumulating the shortage) which minimizes the inventory cost. Inventory depletion is in Fig 1.

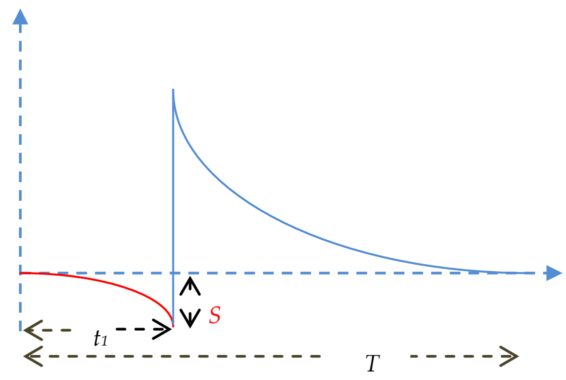

FIG. 1 THE MODEL BEGINS WITH SHORTAGE

The following notations are used:

$D(t)$ demand of product $D(t)=a+bt$, where a and $b>0$ are positive real values,

θ probability of (rate of) deterioration, $0 \le \theta < 1$,

c_1 holding cost unit per unit time,

c_2 shortage cost unit per unit time,

c_3 deterioration cost per unit,

D_C total deterioration cost,

T cycle time,

t_1 optimal time for accumulated shortages,

$C(t_1)$ optimal inventory cost,

D total deteriorated units in system,

S total shortage of units in system,

S_C shortage cost,

H_C holding cost,

Proposed Mathematical Model

Suppose that shortage is denoted by $I_1(t)$ over time t and this continues till t_1. Now management placed the order which immediatly full filled by supplier at t_1 and so on hand inventory is $I_2(t)$. After time t_1 inventory depletes due to demand and deterioration and reduces to zero at time T (see Fig. 1).

$$\frac{d}{dt} I_1(t) = -(a+bt), \text{ where } 0 \le t \le t_1,\ I_1(0) = 0 \quad (1)$$

$$\frac{d}{dt} I_2(t) + \theta I_2(t) = -(a+bt),$$

$$\text{where} \quad t_1 < t \le T,\ I_2(T) = 0 \quad (2)$$

on solving equation (1) we get

$I_1(t) = A - \int_{0_1}^{t} e^{\theta t}(a+bu)\,du$ where A is integral constant which could obtained by boundary condition $I_1(0) = 0$, and so we have $A = 0$.

$$I_1(t) = -\left(at + \frac{bt^2}{2}\right), \text{ where } 0 \le t \le t_1 \quad (3)$$

The negative sign shows the shortage in the system

on solving equation (2) we get

$I_2(t)e^{\theta t} = B - \int_{t_{11}}^{t} e^{\theta t}(a+bu)\,du$ where B is integral constant and could be derived by boundary condition $I_2(T) = 0$, i.e. solution is

$$B = \int_{t_1}^{T} e^{\theta t}(a+bt)\,dt$$

and $$I_2(t)e^{\theta t} = \int_{t_1}^{T} e^{\theta t}(a+bt) - \int_{t_1}^{t} e^{\theta t}(a+bt)\,dt \quad (4)$$

let us expand $e^{\theta t}$ and neglect the higher power of θ, we get

$$I_2(t) = aT + \frac{bT^2}{2} + \frac{a\theta T^2}{2} + \frac{b\theta T^3}{3} - \left(a + a\theta T + \frac{b\theta T^2}{2}\right)t$$

$$+\left(\frac{a\theta - b}{2}\right)t^2 + \frac{b\theta}{6}t^3,\ t_1 < t \le T, \quad (5)$$

deteriorated units in time $(t_1, T]$ is D

$$D = I_2(t_1) - \int_{t_1}^{T}(a+bt)\,dt, \quad t_1 < t \le T$$

$$= \frac{a\theta T^2}{2} - \theta\left(aT + \frac{bT^2}{2}t_1\right) + \frac{a\theta + b}{2}t_1^2 + \frac{b\theta}{6}t_1^3$$

where $t_1 < t \le T$ (6)

Deteriorated cost D_c is

$$D_c = c_3\left\{\frac{a\theta T^2}{2} - \theta\left(aT + \frac{bT^2}{2}t_1\right) + \frac{a\theta + b}{2}t_1^2 + \frac{b\theta}{6}t_1^3\right\}$$

where $t_1 < t \le T$ (7)

Holding cost H_C over time $(t_1, T]$ is

$$H_c = c_1\int_{t_1}^{T} I_2(t)\,dt$$

$$H_c = c_1\left\{\left(aT + \frac{bT^2}{2} + \frac{b\theta T^3}{3} + \frac{a\theta T^2}{2}\right)(T - t_1) - \left(a + a\theta T + \frac{b\theta T^2}{2}\right)\left(\frac{T^2 - t_1^2}{2}\right)\right\}$$

$$+c_1\left\{(a\theta - b)\left(T^3 - t_1^3\right) + \frac{b\theta}{24}\left(T^4 - t_1^4\right)\right\} \quad (8)$$

$$\text{Shortages} = -I_1(t_1) = at_1 + \frac{bt_1^2}{2} \quad (9)$$

Shortage cost S_c is

$$S_c = c_2\int_0^{t_1} I_1(t)\,dt = c_2\left\{\frac{at_1^2}{2} + \frac{bt_1^3}{6}\right\} \quad (10)$$

Number of units including shortage in business schedule are Q

$$Q = I_1(t_1) + I_2(t_1) \quad (11)$$

Total average inventory cost will be

$$T_c = \left\{\frac{H_c + S_c + D_c}{T}\right\}$$

$$T_c = \frac{c_1}{T}\left\{\left(aT + \frac{bT^2}{2} + \frac{b\theta T^3}{3} + \frac{a\theta T^2}{2}\right)(T - t_1)\right\}$$

$$+\frac{c_1}{T}\left\{(a\theta - b)\left(T^3 - t_1^3\right) + \frac{b\theta}{24}\left(T^4 - t_1^4\right) + c_2\left(\frac{at_1^2}{2} + \frac{bt_1^3}{6}\right)\right\}$$

$$+\frac{c_3}{T}\left\{\frac{a\theta T^2}{2} - \theta\left(aT + \frac{bT^2}{2}\right)t_1 + \frac{a\theta + b}{2}t_1^2 + \frac{b\theta}{6}t_1^3\right\}$$

$$-\frac{c_1}{T}\left(a + a\theta T + \frac{b\theta T^2}{2}\right)\left(\frac{T^2 - t_1^2}{2}\right) \quad (12)$$

On equating $\frac{d}{dt_1}C(t_t) = 0$, we get the following equation for optimuml value of t_1

$$-c_1\left(aT + \frac{bT^2}{2} + \frac{a\theta T^2}{2} + \frac{b\theta T^3}{3}\right) - \frac{c_3 b\theta T^2}{2}$$

$$+\left\{c_1\left(a + a\theta T + \frac{b\theta T^2}{2}\right) + ac_2 + c_3(a\theta + b)\right\}t_1$$

$$+\left\{\frac{bc_2}{2} - 3c_1(a\theta - b) + \frac{b\theta c_3}{2}\right\}t_1^2 + \frac{b\theta c_1}{6}t_1^3 = 0 \quad (13)$$

If t_1^* is a positive root of above equation then,

$$\frac{d^2}{dt_1^2}C(t_t) = \left\{c_1\left(a + a\theta T + \frac{b\theta T^2}{2}\right) + ac_2 + c_3(a\theta + b)\right\}$$

$$+\left(\frac{bc_2 - 6c_1(a\theta - b) + b\theta c}{2}{}_3\right)t_1 + \frac{b\theta c_1}{2}t_1^2 \quad (14)$$

At $t_1 = t_1^*$, and for all values of parameters $0 < c_1 < c_2$, $c_3 > 0$, $0 \le \theta < 1$, T and $t_1 > 0$,

$$\frac{d^2}{dt_1^2}C(t_t) > 0, \text{ i.e. } T_c \text{ is optimum} \quad (15)$$

Application and Simulation

Assume that model parameters are a = 20 units, b = 2 units, 1c_1 = \$0.5 per unit per month, c_2 = \$.6 per unit per month, c_3 = \$2 per unit per month, θ = 0.01 and T = 10 days. Under the given parameter values and by using equation (9) to (3) we get output parameters that are t_1=3.901 days, for this $\frac{d^2}{dt_1^2}C(t_t) > 0$ and average total inventory cost $C(t_1)$ = \$486.4, Q = 307 units, average holding cost H_C = \$341.15, Sc=\$103.18.

TABLE 1. VARIATIONS IN PARAMETER

a	b	□	c_1	c_2	c_3	T	t_1	T_c	$S = I(t_1)$	Q	$I_2(t_2)$	H_c	S_c	D_c
20	2	0.01	0.5	0.6	2	10	3.901	486.4	93.24	306.68	213.45	341.15	103.18	42.07
20	2	0.01	0.5	0.8	2	10	3.580	528.2	84.42	307.36	222.94	376.17	114.76	37.34
20	2	0.01	0.5	1	2	10	3.304	562.3	77.00	307.97	230.97	407.49	121.19	33.65
20	2	0.01	0.5	1.2	2	10	3.068	590.4	70.77	308.50	237.73	435.15	124.50	30.76
20	2	0.01	0.5	1.4	2	10	2.862	614.0	65.43	308.98	243.55	459.94	125.62	28.45
20	2	0.01	0.5	1.6	2	10	2.680	634.0	60.78	309.41	248.63	482.33	125.18	26.57
20	2	0.01	0.5	1.8	2	10	2.510	651.4	56.50	309.82	253.32	503.66	122.89	24.94
20	2	0.01	0.5	2	2	10	2.375	666.3	53.16	310.15	256.99	520.79	121.82	23.75
20	2	0.01	0.6	2	2	10	2.649	762.4	60.00	309.48	249.49	583.43	152.74	26.26
20	2	0.01	0.7	2	2	10	2.885	851.1	66.02	308.92	242.90	640.00	182.47	28.70
20	2	0.01	0.8	2	2	10	3.088	933.7	71.30	308.45	237.16	692.44	210.35	30.99
20	2	0.01	0.9	2	2	10	3.267	1011.0	76.01	308.05	232.04	741.20	236.71	33.18
20	2	0.01	1	2	2	10	3.425	1083.8	80.23	307.70	227.47	787.26	261.39	35.23
20	2	0.01	1.1	2	2	10	3.564	1152.8	83.98	307.40	223.42	831.56	284.19	37.12
20	2	0.01	1.2	2	2	10	3.688	1218.4	87.36	307.13	219.77	874.13	305.47	38.88
20	2	0.01	1.3	2	2	10	3.800	1281.1	90.44	306.89	216.45	915.22	325.38	40.53
20	2	0.01	0.5	2	2	10	2.375	666.38	53.14	310.15	257.01	520.89	121.74	23.75
20	2	0.02	0.5	2	2	10	2.455	690.86	55.12	319.91	264.79	523.64	130.38	36.84
20	2	0.03	0.5	2	2	10	2.535	714.86	57.12	329.28	272.16	525.70	139.34	49.82
20	2	0.04	0.5	2	2	10	2.613	738.41	59.09	338.28	279.19	527.28	148.45	62.69
20	2	0.05	0.5	2	2	10	2.689	761.58	61.01	346.94	285.93	528.56	157.58	75.45
20	2	0.06	0.5	2	2	10	2.765	784.35	62.95	355.24	292.29	529.23	166.99	88.13
20	2	0.07	0.5	2	2	10	2.839	806.78	64.84	363.22	298.38	529.61	176.46	100.73

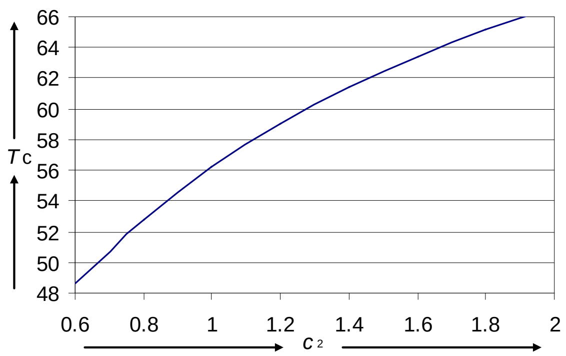

FIG. 2 EFFECT OF C_2 ON OPTIMAL POLICY

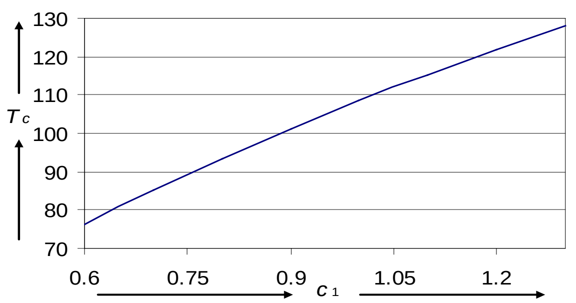

FIG. 3 EFFECT OF C_1 ON OPTIMAL POLICY

Shortage cost highly affects the total inventory cost (fig. 2) up to a level and thereafter total cost is invariant. The optimal time for accumulating shortage is adverse sensitive to this parameter (see table 1) but EOQ is not much sensitive to this parameter. The high holding cost, increases the total cost (fig. 3) and optimal time both but deterioration increases the cost and EOQ both (see table 1). Thus deterioration rate is more sensitive than holding cost and shortage cost is highly sensitive to such type of model especially when shortage occurs in the beginning.

Conclusion

Proposed mathematical model is quite different from the classical model; and provides additional opportunity to managers to place the order according to customer's liking and demand of the product. By minimizing the total cost, the expression of optimal ordering policies are derived.

It is found that shortages and deterioration factors affect the business but shortage cost is more sensitive

than other parameters. To the best possible effort, the inventory managers try to keep low to this parameter. Also managers may negotiate the shortage cost with customer according to assurance to meet the demand at optimal time which was calculated earlier. One can extend the model with variable holding cost and deterioration, with dynamic replenishment. The model could also be formulated in fuzzy environment.

ACKNOWLEDGEMENTS

The author was thankful to referee(s) for fruitful comments which helped to improve the quality of the manuscript.

REFERENCES

Bhaskaran, S., Ramachandran, K., and Semple, J., "A dynamic inventory with the right of refusal", Management Science, 56 (12) (2010): 2265-2281.

Birbil, S.I., Frenk, J.B.G., and Bayindir, Z.P., "A deterministic inventory/production model with general inventory cost rate function and concave production costs", European Journal of Operational Research, 179(1) (2007): 114-123.

Burwell T., Dave D.S., Fitzpatrick K.E., and Roy M.R., Economic lot size model for price dependent demand under quantity and fright discount. "International Journal of Production Economics", 48(2) (1997): 141-155.

Chang, H.C., "A note on the EPQ model with shortages and variable lead time", International Journal of Information Management Sciences, 15 (2004): 61-67.

Emagharby, and Keskinocak P., "Dynamic pricing in the presence of inventory considerations: research overview- Current practice and future direction", Management Science, 49(10) (2003): 1287-1309.

Hou K. L., and Lin L.C., "An EOQ model for deteriorating items with price and stock dependent selling price under inflation and time value of money", International journal of System Science, 37(15) (2006): 1131-1139.

Hou, K.L., "An EPQ model with setup cost and process quality as functions of capital expenditure", Applied Mathematical Modeling, 31(1) (2007) 10-17.

Joglekar, P., Lee P., and Farahani A.M., "Continuously increasing price in an inventory cycle: An optimal strategy for E-tailors", Journal of Applied Mathematics and Decision Science, 8 (2008): 1-14.

Khedlekar, U.K., and Shukla, D., and Yadav M.K., "Pricing policy with time and price dependent demand for deteriorating items", European Journal of Mathematical Sciences, 2(3) (2013a): 341-351.

Khedlekar, U.K., and Shukla, D., "Dynamic pricing model with logarithmic demand", Opsearch, 50(1) (2012): 1-13.

Khedlekar, U.K., "A disruption production model with exponential demand", International Journal of Industrial Engineering Computation, 3(4) (2012): 607-616.

Khedlekar, U.K., and Shukla, D., and Chandel, R.P.S., "Computational study for disrupted production system for deteriorating item", Journal of Scientific and Industrial Research, accepted (2014): to be appear.

Khedlekar, U.K., and Shukla, D., and Chandel, R.P.S., "Logarithmic inventory model with shortage for deteriorating items ", Yugoslav Journal of Operations Research, 23(3) (2013b): 431-440.

Kumar, R., and Sharma, S.K., "An M/M/1/N queuing model with retention of reneged customers and balking", American Journal of Operational Research, 2(1) (2012a): 1-5.

Kumar, R., and Sharma, S.K., "Formulation of product replacement policies for perishable inventory systems using queuing theoretic approach", American Journal of Operational Research, 2(4) (2012b): 27-30.

Kumar, R., and Sharma, S.K., "Product replacement strategies for perishable inventory system using queuing theory", Journal of Production Research and Management, 2(3) (2012c): 17-26.

Lai, C.S., Huang, Y.F., and Hung, H.F., "The EPQ model with shortages and variable lead time", Journal of Applied Science, 6(4) (2006): 755-756.

Matsuyama, K., "The general EOQ model with increasing demand and costs", Journal of Operations Research, 44(2) (2001): 125-139.

Roy A., "An inventory model for deteriorating items with price dependent demand and time-varying holding cost", Advanced Modeling and Optimization, 10(1) (2008): 25-37.

Shin, S.W., "Determining optimal retail price and lot size under day-term supplier credit", Computers & Industrial Engineering, 33(3) (1997): 717-720.

Shukla, D., and Khedlekar, U.K., "Inventory model for convertible item with deterioration", Communication in Statistics-Theory and Methods, Accepted (2014) to be appear.

Shukla, D., Khedlekar, U.K., Chandel, R.P.S., and Bhagwat, S., "Simulation of inventory policy for product with price and time dependent demand for deteriorating item", International Journal of Modeling, Simulation, and Scientific Computing, 3(1) (2010a): 1-30.

Shukla, D., Chandel, R.P.S., Khedlekar, U.K., and Agrawal, R.K., "Multi-items inventory model with time varying holding cost and variable deterioration", Canadian Journal on Computing in Mathematics, Natural Sciences, Engineering & Medicine, 1(8) (2010b): 223-227.

Teng, J.T., and Chang C.T., "Economic production quantity model for deteriorating items with price and stock dependent demand", Computer Operations Research, 32(2) (2005): 297-308.

Wen, U.P., and Chen, Y.H., "Dynamic pricing model on the internet market. International", Journal of Operations research, 2(2) (2005): 72-80.

Wu, K. S., "Deterministic inventory model for items with time varying demand Weibull distribution deterioration and shortages", Yugoslav Journal of Operations Research, 12(1) (2002): 61-71.

You, S.P., "Inventory policy for product with price and time-dependent demand", Journal of the Operational Research, 56(7) (2005): 870-873.

Uttam Kumar Khedlekar works as an Assistant Professor in the Department of Mathematics and Statistics, at Dr. Harisingh Gour Vishwavidyalaya (A Central University) Sagar, Madhya Pradesh, India. He has obtained B.Sc. degree with pure & applied Mathematics, and M.Sc. in pure Mathematics from Rani Durgavati Vishwavidyalaya Jabalpur, Madhya Pradesh, India. He has qualified UGC-CSIR NET Examination in 2006, and till now he has published 17 research paper in national/international journals and one book (Advanced Inventory Models). He has presented five research papers in conferences and workshops. His teaching experience is above 7 years as a permanent faculty and two year at the college level as a guest faculty. He is editorial board member in journal "Uncertain Supply Chain Management" (Iran), and reviewer of journals AJMO, ASOR, BJEMT & IOSR journals.